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# Chapter 1: Schedule & Program

# IMERS 2021 Schedule

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# Program

# 1st Session (9:25-10:10)

## Completed study

**Developing Better Instruction, Better Instructors, and New Investigators**
*Dr. Patrick Eggleton, Dr. Barbara Johnson, Clay Vander Kolk, and Abby Pyle*

This research discusses how a group of HAMTE educators collaborated to use the Principles to Action Professional Learning Toolkit in their classes, the tools they used to measure development in the future teachers, and the research results that were developed by future mathematics teachers. This entailed a survey from future educators in a "mathematics for elementary teachers' course, the statistical analysis by two future secondary math educators, as well as the observations and conclusions they made that expand on the effectiveness of the NCTM Toolkit and the implications this can have for all future educators as well as current seasoned teachers.

## Work in-Progress

**Examining Tall's Worlds in Curriculum**
*Kelsey Walters*

Multiple representations of mathematical objects play a key role in undergraduate linear algebra courses. These representations may be classified in terms of Tall's worlds of mathematics: the embodied world, the symbolic world, and the formal world. Harel has examined how the embodied world appears in textbooks and the purposes for which it is used. In this study, I will extend this work by qualitatively analyzing how each of the three worlds appears in the intended, enacted, and attained curricula by examining selected portions of the textbook, lectures, and student work from a linear algebra course. Preliminary results from the textbook analysis indicate that the embodied world is used primarily for generalizations and literal representations. Beyond definitions, theorems, and proofs, the formal world is used for non-proof explanations, preventing misconceptions, and introductions. The symbolic world has the widest variety of uses, including all the ways the formal world is used as well as providing computational procedures and connecting the material to computational issues with technology. The ways worlds are used to introduce concepts is not constant. When introducing linear independence, the textbook moved from the symbolic world to the embodied world. The introduction of span started and ended in the embodied world, with the symbolic world in between. Further analysis will refine these results and determine whether the worlds appear in similar ways and are used for similar purposes in the intended, enacted, and attained curriculum.

**A Comparative analysis of Congruence in U.S and Chinese Standards and Textbooks**
*Lili Zhou, Jane-Jane Lo, and Jinqing Liu*

Study reveals that U.S. and Chinese curriculum use different approaches to define congruence and address the topic differently (Zhou et al., under review). This study aims to examine the topic of congruence in the U.S standards and Chinese mathematics standards and textbooks. We investigate Common Core Mathematics State Standards (CCSSM) which is widely adopted in the U.S and Chinese Compulsory Education Mathematics Curriculum Standards (CMCS) which is the only national standards for elementary and middle school in China. For comparison, we selected the widely used Chinese textbook for Grade 8, published by People Education Press, which is aligned with CMCS, and the U.S. textbook for Grade 8 – Eureka Math, published by Great Minds, which is aligned with CCSSM. The present study is guided by two overarching research questions. First, how CCSSM and CMSC present the topic of congruence and what learning expectations are proposed in these two sets of standards. Second, how the two textbooks proceed the concept of congruence and congruent triangles. The ongoing analysis indicates that both textbooks refer to congruence as same size and same shape. However, in line with the standards in both countries, the two textbooks present congruence in different ways.

**Examining students' understanding of numbers and operations using an online number sense three-tier test**
*Iwan A.J Sianturi, Zaleha Ismail, & Der-Ching Yang*

Numbers and operations typically occupy a large time of instructions and learning contents in elementary schools, given its importance in providing the fundamental mathematics concepts and skills. This study documents 372 fifth-grade students' understanding of numbers and operations when responding to an online three-tier number sense test, which examined content knowledge (i.e., questions and answers), explanatory knowledge (reasoning/reasons), and confidence level regarding the answers and reasons.

**Understanding an Undergraduate's Units Coordination**
*Patti Walsh*

Units coordination and spatial reasoning have been investigated with elementary and middle school students to support better instruction in those areas, as they can impact future mathematical learning in higher grades. But what happens when a student reaches college without having being able to coordinate three levels of units and without a robust conceptual understanding of fractions? This teaching experiment explores such an undergraduate's thinking and looks for mathematical tasks that might support the development of more sophisticated understanding of units coordination and fractions for that student.

# 2nd Session (10:20-11:05)

## Completed Study

**Gestural Scaffolding: A first-grade preservice teacher's enactment of probes**
*Lizhen Chen*

Many studies have focused attention on inservice teachers' gestural use and comprehension; little attention has been paid to preservice teachers. Contrary to a claim that preservice teachers know little about gesture use, this study supports with evidence the assumption that the PST was able, consciously or not, to utilize various gestures along with speeches to accommodate their probing intent as well as students' cognitive demands especially in situations of students' struggles. In this case study, I collected a preservice teacher's teaching video, her identification of probes in a stimulated recall interview, and researchers' identification of probes. In data analyses, I provided an overview of her probes implemented with speech and gesture and compared her identification of probes with researchers' identification. Further implications about preservice teacher education are also discussed in the end.

## Work in-Progress

**Finding Limit-Situations, Moving Towards Limit-Acts: Women Narratives in Graduate Mathematics Programs**
*Weverton Ataide Pinheiro*

Gender issues in mathematics education received fairly attention several decades ago, mainly because women and men performed differently, with men usually outperforming women. The research on such phenomena originated a series of studies on the so-called 'achievement-gap.' After many years of research on the achievement gap, such a gap has shown to have disappeared. However, recent studies have shown that there are fewer women who choose STEM-related fields than male peers. Suppose women are not choosing mathematics, but concomitantly research has pointed out that women and men have similar mathematics capabilities; why are women disproportionately part of the field? Since current statistics show that 90-95% of doctoral mathematics students are men, this study examines why school mathematics is still a male-dominant field. Also, to what extent the survival of women's mathematics graduate school students' experiences foregrounds the need for women's non-extinction in the field.

**Confronting Whiteness: An Action Research Study of a Social Justice Mathematics Course**
*Michael Lolkus*

Mathematical tasks that address social justice issues have shown promise for bringing attention to the voices and experiences of people of color, supporting students' critical mathematics development, and facilitating socio-political understandings. While teaching mathematics for social justice can support students to develop critical mathematics literacy – the specific understandings about how mathematics can be used to determine whose knowledge is valued –

efforts to deliver equitable, social justice-oriented, mathematics instruction still run the risk of perpetuating whiteness. As such, my study details a necessary reflection of my own positionality as a white cis-gender male and consequential perpetuation of whiteness throughout Knowing the World Through Mathematics, a social justice mathematics course I designed and taught in Fall 2020 to 11 prospective mathematics teachers. In this action research study, I utilize thematic analysis to organize themes across various sources of evidence, including instructor reflections (i.e., weekly, pre-course, post-course) and curricular documents (i.e., lesson outlines, mathematics tasks, syllabus). This action research study will provide further insight to how social justice mathematics can implicitly perpetuate whiteness, and how mathematics teachers and teacher educators can work to confront and decenter whiteness in their respective classrooms.

## Work under-Design

### Teachers' care behind the stage: Teachers Curricular Decisions
*Ana-Maria Haiduc*

Mathematics builds on itself, but this cumulative creation in students' minds requires careful selection of problems. Multiple studies linked teachers' conceptions, skills, and goals to curriculum decisions, less research explores teachers' care concerning curriculum decision making. This case study investigates how teacher's care influences curricular decisions. Data from 10 teachers consisting of a questionnaire, interview, curricular resources considered and selected will be analyzed using the caring mathematically framework. Findings in the form of correspondence between traditional factors that influence teachers' curricular decisions and ways teachers harmonize with the students' thinking to analyze the curricular used will be constructed. These findings support the argument that teachers’ care is a factor in their curricular decisions.

### Complexity Theory and Virtual Education. What is it changing?
*Jonathan Rojas Valero*

This proposal corresponds to a research process in its initial state, i.e. work under design. Firstly, a rational to investigate virtual mathematics classroom is defined. Secondly, the study considers a literature review in relation to the Complexity Theory in education, particularly, considering the five conditions to understand the classroom as a complex system (Davis & Simmt, 2003): internal diversity, redundancy, decentralized control, organized randomness, and neighbor interactions. Lastly, research questions and possible methods are proposed.

# 3rd Session (12:10-12:55)

## Work in-Progress

**Core Practices in the Context of an Elementary Mathematics Methods Course**
*Patti Walsh*

Paired with learning theory, core practices can help pre-service teachers gain practical knowledge that they can use to effectively implement instruction. This self-study looks at how the core practices of organizing class discussion and eliciting student thinking are represented, decomposed, and approximated in the context of an elementary mathematics methods course.

**Preservice Teachers' Use of Mathematical Knowledge for Teaching During Lesson Planning**
*Bima Sapkota*

Teacher education programs need to provide mathematics preservice teachers (M-PSTs) with opportunities to conceptualize how content knowledge is utilized in secondary mathematics teaching. However, M-PSTs have limited opportunities to conceptualize how the content knowledge learned from university-level mathematics courses is contextualized in mathematics teaching, including lesson planning. In this study, I investigate how Mathematical Knowledge for Teaching influences M-PSTs' lesson planning skills. A group of M-PSTs enrolled in a secondary mathematics methods course at a large midwestern university participate in this study. I design a series of instructional activities in which M-PSTs have opportunities to engage in two rounds of lesson planning, lesson implementation, and reflection on lesson planning and implementation. M-PSTs also have opportunities to share their reflections with their peers. As an implication of the study, teacher educators can utilize the instructional activities in teacher education programs to provide M-PSTs with opportunities to conceptualize how content knowledge is utilized during lesson planning.

## Work under-Design

**Angle your laser: Technological exploration of using radians in a STEM context**
*Hanan Alyami*

This study aims to explore the application of radian angle measure in a novel context. Specifically, exploring PSTs ways of thinking about the concept of radian angle measure within the scientific concept of light reflection. This study is informed by the overall stance of reforms supporting the integration of science, technology, engineering, and mathematics [iSTEM] since project 2061 (e.g., American Association for the Advancement of Science, 1989; National Council for Teachers of Mathematics, 2000; National Council of Supervisors of Mathematics (NCSM) and National Council of Teachers of Mathematics (NCTM), 2018; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Research Council [NRC], 2013; Next Generation Science Standards Lead States, 2013). In this study, iSTEM is conceptualized as the intentional and purposeful interaction between different disciplines to merge their disciplinary perspectives (Wang, Moore, Roehrig, & Park, 2011). Such approach aims to encourage a meaningful interaction of the ontology and epistemology of at least two disciplines (e.g., science and mathematics, mathematics and technology, science and mathematics and engineering, etc.), to achieve meaningful learning. The increased recognition, adoption, and implementation of iSTEM curricular approaches increase the need for teachers who are prepared to implement iSTEM activities (NRC, 2013). With all these recommendations, the NRC (2013) has acknowledged the importance of curricular development and understanding of teachers' knowledge about the concepts that are involved in iSTEM instructional activities. This study is guided by the research question "What conceptions about angle measure are expressed by PSTs upon interacting with an online iSTEM exploration activity that involves radian angle measure? "

**Exploring the combinatorial reasoning in undergraduate students' classrooms**
*Iris Mariela Duarte Mejia*

The study proposed addresses the teaching and learning of combinatorics; particularly, its basic counting strategies and formulas. As different researchers have studied individual student's reasoning and learning of combinatorics, this proposal aims to analyze the development of combinatorial thinking when is being studied in the context of a regular undergraduate course.

**Impact of units coordination in reminding fractional knowledge for high school students**
*Hyunjeong Lee*

In my experience teaching math for high schoolers, I had many students having difficulty in calculating fractional numbers, even though they learned fractions since they were in elementary school. In order to help those students, teachers need to lead their students to consider fractions as quantity. In this regard, units coordination can be a solution to do that with partitioning, disembedding, and iterating. In this proposal, I would like to show that students with a lack of fractional knowledge can improve and robust their fractional knowledge by learning partitioning, disembedding, and iterating as units coordination.

**Exploring Pre-Service Teachers' Lesson Plans to Promote Improper Fractions**
*Selim Yavuz & Sezai Kocabas*

Splitting operation is an essential skill to produce improper fractions. Students can have splitting operations with/without interiorizing three levels of units. Therefore, pre-service teachers can be able to design lesson plans involving different tasks for all students. In this study, we will explore pre-service teachers' pedagogical content knowledge by analyzing their lesson plans on improper fractions.

# 4th Session (1:05-1:50)

## Work in-Progress

### Understanding the Mathematical Experiences of Latinx STEM Majors: A Rehumanizing Approach

*Alexandria T. Cervantes*

In this proposal, the author presents a study focused on understanding the mathematical experiences of Latinx STEM(science, technology, engineering, and mathematics) majors at a predominantly white university in the Midwest. The author aims to employ Gutierrez's Rehumanizing Mathematics as a theoretical perspective and counter storytelling to analyzing how these Latinx STEM majors navigate, understand and experience mathematical spaces, along with their sense of belonging in the subject. This exploratory study will help illuminate what is currently happening in the mathematical spaces found at a university (i.e. mathematics classrooms, study groups, and office hours), along with demonstrating what a rehumanized undergraduate experience could look like. These findings have implications on how future undergraduate mathematics education courses could be restructured to broaden and reshape mathematics for future students.

### Image of Mathematics in and out of School: A Case Study of a Group of Original Girls Participants in an Afterschool STEM club

*Rose Mbewe & Lili Zhou*

In public, mathematics is viewed as abstract, cold, and irrelevant to real-life. Previous studies have shown that one's school experience greatly influences their image of mathematics. The present study investigates a group of women (n=9) who participated in an afterschool school STEM club 26 years ago and aims to seek how their described experiences in school and in the informal learning environments contribute to their images of mathematics. The data were collected from survey, focus group interviews, and individual interviews. We use thematic analysis method to interpret the data. The finding shows some aspects of informal learning environment which potentially change these women's image of mathematics. The implication of the study suggests feasibility to improve mathematics learning environment that mediate current public image of mathematics.

## Work under-Design

### Going Beyond Gender-Complex Education: A Study of Sexual Orientation in Mathematics Education

*Weverton Ataide Pinheiro*

This study pushes back on the teaching of mathematics for social justice and studies that ascertain issues of gender overlooking issues of sexual orientation. Although the author sees an extreme importance of the teaching of mathematics for social justice, and this is the author's ultimately desires (that the teaching and learning of mathematics always help students to challenge social issues), it's hard to know and write about what is effective and what will produce positive engagement towards learning, if students from these different genders and sexual orientations aren't consulted at first. I am not arguing that no research has been done by mathematics educators in trying to understand transgender and gender-nonconforming students' experiences, but there is a trend after Rands (2013) publication on Add-Queer-And-Stir and Mathematics Inque(er)y that have resulted in multiple investigations of how to queerize the curriculum. Interestingly, little in this new realm of research has considered the students' experiences. Rather, they suggest what can be done to make the classroom more inclusive to LGBTQ+ students or dismantle a variety of normativities that come in mathematical problems. Because of the complexity of the LGBTQ+ groups, and having in mind that such an acronym refers to gender identities, sexual orientation, and sexual identities, research has also overlooked the sexual orientation part of these groups. Leaving gays, lesbians, bisexuals, asexuals, demisexuals, and graysexuals excluded from important conversations that must be challenged as mathematics continues to propagate heteronormativity. In addition, not respecting and understanding how we can in fact impact learning for LGBTQ+ students in ways that we are going forward creating safe learning spaces, in fact, for these students. Thus, this proposed study will investigate how LGBTQ+ students respond to issues of sexual orientation being discussed in the mathematics classroom.

### Exploring the Impact of Mathematics Problem Solving as Informal Play in the Time of COVID -19

*Ana-Maria Haiduc*

The curiosity for mathematics is delightful in informal settings. This research explores people's involvement in mathematical play activity during COVID-19 quarantine. An easel placed near the sidewalk in an urban subdivision showed mathematics problems during the COVID-19 pandemic. People embraced with joy this social activity in the form of mathematics play during the quarantine. Interviews with eight participants in the mathematical activity will reveal what they have experienced and how they experienced the mathematical activity as adults after they have finished school.

# Chapter 2: Completed Studies

**Gestural Scaffolding: A first-grade preservice teacher's enactment of probes**

Lizhen Chen
Purdue University

**Abstract**

*Many studies have focused attention on inservice teachers' gestural use and comprehension; little attention has been paid to preservice teachers. Contrary to a claim that preservice teachers know little about gesture use, this study supports with evidence the assumption that the PST was able, consciously or not, to utilize various gestures along with speeches to accommodate their probing intent as well as students' cognitive demands especially in situations of students' struggles. In this case study, I collected a preservice teacher's teaching video, her identification of probes in a stimulated recall interview, and researchers' identification of probes. In data analyses, I provided an overview of her probes implemented with speech and gesture and compared her identification of probes with researchers' identification. Further implications about preservice teacher education are also discussed in the end.*

## Introduction

Many studies (e.g., Alibali, et al., 2019; Nathan, et al., 2017) have focused attention on inservice teachers' gestural use and comprehension; little attention has been paid to preservice teachers (PSTs) who are still learning to teach mathematics and manage their own expressions and displays of meanings. Prior studies on inservice teachers' use of probes have been dominated by the powered lenses of researchers while leaving little space for the teachers' voices of their own probing experiences. Therefore, this study aims to first present PSTs' voices in their enactment of probes, particularly addressing the issue of how gesture scaffolded communicative meaning-making; and second delve into differences between PSTs' identification of probes and researchers' identification.

## Theoretical Framework

### Gestural Grounding

Gesture accompanies speech and contributes to meaning making by conveying similar or additional information (McNeil, 2017). Gesture in action (e.g., deictic gesture) interacts with concrete materials in the context; gesture in verbal communication (e.g., representational gesture) functions as an imitation of action and thus takes on certain selective features of movements so that listeners can make sense of what is represented (Clark & Gerrig, 1990; Gerwing & Bavelas, 2004). An example of selective movements is the gesture of counting by ones (i.e., pointing to imaginary dots one by one in the air). Its sequenced points and regular stops in-between two adjacent imaginary points are symbolic of counting concrete objects one by one, therefore signaling listeners the messages speakers intend to convey.

Gesture, when grounded in physical contexts such as objects and actions, facilitates students' access to the meaning embedded in speech (Alibali & Nathan, 2007). Common ground refers to the shared information between speakers and listeners (Alibali et al., 2013; Gerwing & Bavelas, 2004; Holler, 2009). In classroom instruction, when common ground is lost between teachers and students, confusion rises up among students and teachers find it necessary to slow down their instruction and apply more verbal and non-verbal signs to assist students in learning. The teacher is likely to increase their use of grounding gesture in trouble spots (Alibali & Nathan, 2007) and in the introduction of new information (Gerwing & Bavelas, 2004). Deictic gesture is frequently used in mathematics classrooms when teachers connect speech with visual representations (e.g., drawings, diagrams). Gerwing and Bavelas (2004) also reported the importance of gesture use in communication.

## Methods

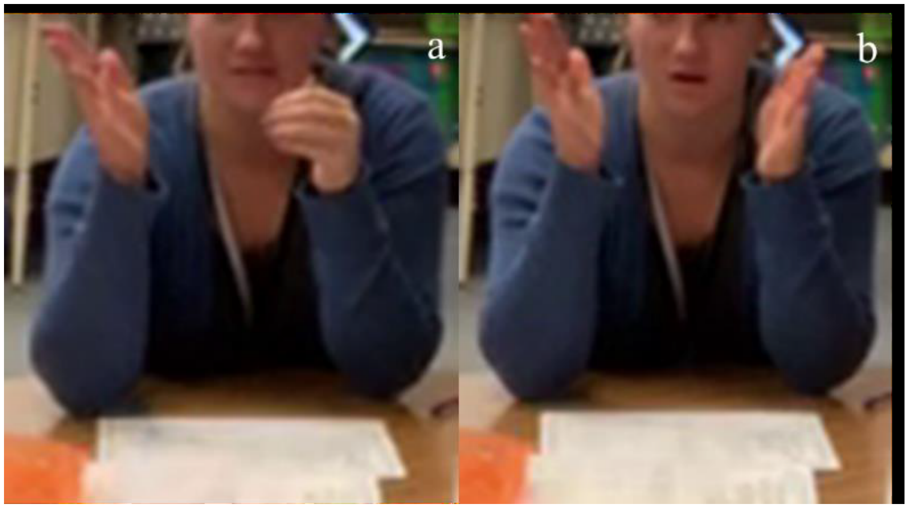
This study examines Clara's, a PST, use of probes in a Grade 1 classroom. Clara was taking an elementary mathematics course as required by the teacher certification program in a Midwest university. After doing lesson planning in the methods course, Clara implemented her lesson plan about two-digit number comparison in an urban elementary school. Her lesson targeted the facilitation of meaningful discussions and lasted 39 minutes. Clara's teaching was videorecorded. Afterwards, a stimulated recall interview was conducted with Clara in order to record Clara's understanding of her own probe use. Finally, two researchers independently coded Clara's teaching video and discussed their codes of probes till 100% agreement was achieved.

## Findings and Implications

Overall, Clara's variation of probes grounded in gesture helped students practice using the mathematical language, helped students dig into their perception of place value, and provided them with opportunities to sharpening adaptive reasoning. A case in point is when Clara probed students' place-value understanding. In this probe, Clara intended to probe a student's thinking about the place value, i.e., which place is bigger between 58 and 59. She first probed, "What's bigger?" with a deictic gesture (pointed to the problem on the paper worksheet) and found out the student understood her probe as "Which is bigger, 58 or 59?" Then she revised her probe as a combination of "What does it have more of?" with the same deictic gesture. The student incorrectly answered that "It has more of ten." Taking a step further, Clara probed by asking "What does it have more of? Is it ten or one?" with a metaphoric gesture (put right hand vertically up, then left hand vertically up), thus making her probe objectively accessible to the student by restricting her probe within two choices. The metaphoric gesture with two hands vertically up abstractly represents ten and one as if right hand stands for a side for ten and left hand for the other side for one. Clara confined her probes by means of asking the student to choose a side to stand by; she grounded these two choices in two hands and made it accessible to the student. One the one hand, the metaphoric gesture bears a deictic aspect, referring to ten and one [gesture is imaged based (McNeill & Duncan, 2000)]; on the other hand, the metaphoric gesture represents two solutions/concepts/opinions, i.e., whether 59 has more of tens or more of ones than 58 (place value). Despite the student's failure to answer what 59 has more of, Clara herself constantly refined her speech and used increasing grounding gesture to better probe student thinking.

**Figure 1**

*Place-Value Probe with a Metaphoric Gesture Representing Ten and One*




When probing mathematics concepts (i.e., place value and comparison signs), Clara was consistent with her language use throughout her lesson, which contributed to her high alignment with researchers' coding of probes. There are two instances when Clara missed her probes: first, she did not count a concept probe with one student and instead counted the same probe with another student that followed tightly the previous probe; second, she missed out one concept probe occurring within eight seconds. Besides, Clara identified instances of digging into students' experience with mathematics as mathematics probes; whereas, researchers did not identify these instances as probes. For example, at the beginning of each group interaction, Clara inquired into students' experience of getting to know comparison signs (i.e., "How do you know that?") and one student responded with his childhood stories.

To sum up, Clara varied her gestures when encountering students' struggles with understanding of her questions; varied gestures and questioning speech consist of her probes characterized with flexibility and richness in meaning-making. The slight differences between Clara- and researcher-identified probes imply Clara's clear awareness of her probing intent. These findings contribute in important ways to the enactment of probing practices in preservice teacher education.

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# Developing Better Instruction, Better Instructors, and New Investigators


| Patrick Eggleton | Barbara Johnson | Clay Vander Kolk | Abby Pyle |
|---|---|---|---|
| Taylor University | IUPUI | Taylor University | Taylor University |



## Abstract

*This research discusses how a group of HAMTE educators collaborated to use the Principles to Action Professional Learning Toolkit in their classes, the tools they used to measure development in the future teachers, and the research results that were developed by future mathematics teachers. This entailed a survey from future educators in a "mathematics for elementary teachers' course, the statistical analysis by two future secondary math educators, as well as the observations and conclusions they made that expand on the effectiveness of the NCTM Toolkit and the implications this can have for all future educators as well as current seasoned teachers.*


In the training of teachers for the classroom, there has been an ongoing question of how to help future teachers move from the "teacher-centered" focus of instruction to a more "student-centered" focus of instruction. The National Council of Teachers of Mathematics (NCTM) recently published a series of resources called the *Principles to Actions Professional Learning Toolkit* that provides classroom videos that allow viewers the opportunity to view "student-centered" lessons that promote recommended teaching practices for mathematics. The purpose for our research is to do three things: Implement use of the Principles to Action Toolkit with other Mathematics Teacher Educators (*Developing Better Instruction),* document our results by measuring any changes in commitments toward effective mathematics teaching practices *(Developing Better Instructors),* and lastly, involve undergraduate mathematics education majors in the analysis of the resulting data *(Developing New Investigators).* We wanted to find, "does exposure to video examples of student-centered mathematics instruction in elementary classrooms contribute to student's commitment to this type of instruction?"

Our research provided a pre/post class survey that could measure future teachers' commitment toward using student-centered type instruction in their future mathematics classes. To gather our data, an electronic survey with two open-ended questions and 42 questions requiring a choice of expected instructional use was given to students at the beginning of a "mathematics for elementary teachers" course. Completion of the survey was voluntary for students and not connected to any course requirement. During the semester the course instructor included at least 5 of the NCTM Professional Learning Toolkit videos as a required component in the course. At the end of the course, students were asked to complete the follow-up electronic survey with the same questions as the initial survey with an additional open-ended question where students shared any factors that contributed to a change in beliefs about teaching (if such a change took place).

With this data, and after provision of a grant in Fall 2020, Abby Pyle and Clay Vander Kolk, junior mathematics education majors at Taylor University, analyzed both the quantitative and qualitative data. As future math educators, they were able to use their statistical knowledge to run tests and interpret the quantitative data as well as code the qualitative data to run tests, make observations and ultimately some conclusions that could support the usage of the NCTM toolkit in future classrooms. Through this analytical work, they found it appears that there was a general movement/trend from more teacher-centered, transmission-based teaching to more student-centered, constructivist-based teaching as well as the PSTs ideas had changed, as they reported growth or desire towards implementing many of NCTM's Effective Mathematics Teaching Practices. There is a lot of future studies this could be applied to and can really help all students see the benefit and effectiveness of alternate teaching strategies and using tools and teachings strategies that may be foreign. The conclusions in this data also can guide all teachers, regardless of experience, in checking themselves along the teacher-centered to student-centered "continuum". Also, because of the nature of Pyle and Vander Kolk's enrollment at Taylor University, where this study took place, they were able to get a unique view at the change, behavior, and feedback from the students while also reflecting back on a similar class with the same professor. Throughout this research, they were able to make reflect, engage in mathematical concepts, as well as apply this to not only their own teaching philosophies but share their ideas to mathematicians everywhere.

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# Chapter 3: Work in-Progress

**Understanding the Mathematical Experiences of Latinx STEM Majors: A Rehumanizing Approach**

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Although mathematics education has experienced a shift to focus on "equitable practices," (Gutierrez, 2013) the current mathematics education model in the United States still tends to privilege a banking model of education(Freire, 2018) that encourages students to "play the game" of mathematics, including obtaining high tests scores, mastering content, and participating in the STEM pipeline, rather than "changing the game" which includes changing who is seen as good at mathematics, understanding power dynamics and broadening what counts as mathematics(Gutierrez, 2009 & 2012) . This limitation of focusing on playing the game rather than changing the game often serves to dehumanize students and impacts their outlook and overall relationship with mathematics inside and outside of the classroom.

When attempting to address the inequities in STEM amongst Latinx students, studies have primarily used quantitative analyses to capture large scale views of students(Landivar, 2013), especially at Hispanic Serving Institutions(Crisp et al, 2015). We know little about Latinx students in predominantly white institutions and from a qualitative perspective that could more accurately capture concepts like belonging or what needs to change about mathematics as a discipline. When such studies have had a qualitative focus, they have primarily been conducted in K-12 settings or focus on students who are Black or Asian(McGee & Martin, 2011). My research aims to fill the gap in the field to understand Latinx students' humanizing and dehumanizing experiences and apply the Rehumanizing Mathematics framework to address undergraduate mathematics.

I use Gutiérrez's Rehumanizing Mathematics(2018) framework to guide my research. Gutiérrez details eight practices for Rehumanizing Mathematics, including changing the positioning of authority in classrooms, incorporating cultures and histories into mathematics, and recognizing mathematics as a living practice. I use the term humanizing to embody these eight terms, which also include broadening participation in mathematics, using mathematics as a way to see a reflection of oneself and as a window to view into other people's lives and experiences, creation in math, understanding how our bodies and emotions can play a role in mathematics along with having ownership over the subject. I use dehumanizing to frame instances that would cause students to have negative tensions with these practices.

The research questions guiding my study are 1) What are the ways Latinx students have humanizing and dehumanizing learning mathematics experiences in K-16 spaces? 2) How do those experiences influence their identities and conceptions of agency?

The work-in-progress study will be presented as a case study, where I will focus on two students, Anastasia and Isabella, and their experiences as Mexican American and Puerto Rican American STEM students. Anastasia speaks to how her sense of community and belonging impacted how she navigated challenging coursework and the politicalness of studying mathematics at a predominately white institution. Second, Isabella describes how dehumanizing experiences can affect the academic performance and trajectory of students and how these can impact the way students see themselves in relation to mathematics and as doers of mathematics.

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# Confronting Whiteness: An Action Research Study of a Social Justice Mathematics Course

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While much research has moved to investigate whiteness in mathematics education (e.g., Battey, 2013; Harper et al., 2020; Martin, 2009; Stinson, 2011), scholars have yet to turn the lens of whiteness upon their own instructional practices and curricula. I seek to analyze how *Knowing the World Through Mathematics* (*KWM*)*,* a mathematics content course I designed to address issues of social justice is framed by, and unintentionally perpetuates, whiteness. Given the need for ongoing efforts to decenter whiteness and produce culturally relevant pedagogies, I detail an action research study (Kemmis et al., 2014) on how a critical investigation of whiteness (Frankenberg, 1993) in a social justice mathematics course, *KWM*, informs (a) my work as a mathematics teacher educator; and (b) revisions of *KWM*.

## Methods

Drawing from action research (Kemmis et al., 2014) and the qualities of critical research in mathematics education (i.e., *current situation*, *imagined situation*, *arranged situation*; Skovsmose & Borba, 2004), I reflect on and confront my own complicity in perpetuating whiteness in the *KWM* curriculum. As I engage in a critical reflection of my instructional practices and curriculum design, I will also work to take action to revise the curriculum to more thoroughly challenge the implied racial hierarchies in mathematical spaces.

### Sources of Evidence

Using action research methods (Kemmis et al., 2014), I critically reflect on my own perpetuations of whiteness, or "the unwillingness to name the contours of racism, the avoidance of identifying with racial experience or group, the minimization of racist legacy, and other similar evasions" (Frankenberg, 1993, p. 23), in the curriculum and instruction of *KWM*. Evidence for this action research study consists of my own unstructured, voice recorded memos from the end of each *KWM* lesson, pre- and post-course semi-structured written reflections, as well as the intended *KWM* curriculum.

### Evidence Analysis

I am engaging in thematic analysis of all evidence sources (i.e., unstructured reflections, semi-structured reflections, intended curriculum). Thematic analysis, which refers to investigating and organizing common themes across evidence sources, allows me to explore themes across a variety of data sources (Braun & Clarke, 2012). My analysis is guided by a critical perspective (Skovsmose & Borba, 2004), as well as the *institutional* and *labor* dimensions of Battey and Leyva's (2016) Framework for Whiteness in Mathematics Education.

#### *Establishing the Quality of Research Design*

Following Nowell et al.'s (2017) recommendations for trustworthiness in thematic analysis, I rely on my undergraduate research partner, Gabrielle Gagnon, to provide additional perspectives for adequate triangulation of findings (Flick, 2018). Furthermore, I am maintaining detailed documentation of the coding and debriefing processes throughout each phase of thematic analysis in reflexive journals of all researchers (i.e., Gagnon and myself). Checking my themes and analyses with Gagnon encourages consensus of themes before I disseminate findings (Nowell et al., 2017).

**Scholarly Significance**

This action research study informs ongoing efforts to decenter whiteness in mathematics education through a critical reflection of implicit connections and perpetuations of whiteness in a social justice mathematics course.

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# Finding Limit-Situations, Moving Towards Limit-Acts: Women Narratives in Graduate Mathematics Programs

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Although women's representation in mathematical spaces is still relatively lower than that of their male counterparts, in recent years, research on gender and mathematics has not been given enough attention by researchers even though it should (Lubienski & Pinheiro, 2020). In the late 1990s and beginning of the 21st century, with the advancement of research on Gender Studies and a better understanding of the meanings of gender, mathematics research followed suit, and researchers attempted the usage of new theoretical standpoints to conduct research. In such studies, rather than comparing gender through objectivist lenses, interpretive methodologies were taken into consideration (see Leyva, 2017).

Recent studies have attested a variety of reasons why women are still underrepresented in mathematics, such as mathematics is a masculine field (Mendick, 2005), women feel isolated (Herzig, 2004; Johnson, 2011), and sense of not belonging (Herzig, 2010; Solomon, 2007). However, little research has attempted to uncover the conditions that women are experiencing in mathematical spaces (*limit-situations* or barriers) and proposing ways for women to reflect on their situation and move forward (*limit-act* (s): appropriate action) (see Freire, 2017). In particular, this research proposes to answer the following questions:

1. What are the limit-situations experienced by women in mathematics graduate programs, and what should we do about them?

**Perspective(s) or theoretical framework**

Researching gender is particularly hard due to the complexity in the definition and theorization of gender (Esmonde, 2011). This study sees gender as a socially constructed apparatus that is both fluid (not fixed categories such as sex) and performative (Butler, 1990). Through equity lenses, the motto for gender research lies in the unbalanced and disservice women experience in places of underrepresentation. Mainly, underrepresentation is a long-vivid experience for women in mathematical spaces. Because of underrepresentation, women's experiences in mathematics are conditioned as oppressive.

*The Pedagogy of the Oppressed* provides theoretical underpinnings for understanding the lives of people conditioned by oppression and a methodological way to research oppressed groups. Oppression is seen as a condition established between the oppressor and the oppressed (Freire, 2017). Hypothesizing women's experiences is conditioned to their gender; this study attempts to seek the limit-situations women experience in mathematics by analyzing contradictions in these women's experiences. I follow Chauí's (1982) definition of contradictions, which means the unity and struggles of opposites.

**Methods and Methodology**

This study took place in the mathematics department of a large Midwest research institution (pseudonym: Reed University). High achievers mathematics graduate students, who identified as women and were between the 1st and 6th year of the Ph.D. program were recruited. This study data was collected through semi-structured interviews. The 2 two-hour interview data were informed by *life stories interview* approach (McAdams, 2008).

**Findings and Conclusion**

This study found that the field of mathematics has made a great achievement in terms of "who is counting" and who is part of it. In particular, women still go through hard times in the mathematics department, and they still point to facts in the department that alienate their experiences. This research was relevant because it uncovered the current situation women graduate students go through in *mathematical spaces*. Both positive and negative aspects will be shared during the presentation in IMERS.

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# Image of Mathematics in and out of School: A Case Study of a Group of Original Girls in an Afterschool STEM Club

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**Abstract**

*In public, mathematics is viewed as abstract, cold, and irrelevant to real-life. Previous studies have shown that one's school experience greatly influences their image of mathematics. The present study investigates a group of women (n=9) who participated in an afterschool school STEM club 26 years ago and aims to seek how their described experiences in school and in the informal learning environments contribute to their images of mathematics. The data were collected from survey, focus group interview, and individual interviews. We use thematical analysis method to interpret the data. The finding shows some aspects of informal learning environment which potentially change these women's image of mathematics. The implication of the study suggests feasibility to improve mathematics learning environment that mediate current public image of mathematics.*

Mathematics is one of the fundamental bases of an individual's school formation and presents some features independent of the cultural context and territorial factor (Pattison et al., 2017). From the public point of view, mathematics is perceived as "difficult, cold, abstract, and in many cultures, largely masculine" (Ernest, 1996, p.802). Researchers argue that anyone who has had some schooling has an image of mathematics present in them (e.g., Nunes & Bryant, 2010). Furinghetti (1993) argues that an adult's image of mathematics, whether positive or negative, is formed and conditioned by the individual's school experience.

School mathematics tends to prepare students for standardized tests that focus on providing isolated instruction with limited opportunities for students to connect mathematical ideas which might result negative image of mathematics (Copper, 2011). In the classrooms, the gaining of techniques and manipulative skills is given priority despite being less rewarding to the students. Students' attention and effort concentrate on it because it is usually at this stage that learning is evaluated (Civil & Andrade, 2002).On the other hand, informal learning environments are settings where rich mathematical thinking and reasoning outside the classroom can occur. Unlike schools, these settings offer individuals and groups the opportunity to more freely choose how, what, where, and with whom they learn (Falk & Dierking, 2013). Therefore, informal learning spaces have been suggested as an alternative place, potentially mediate negative image of mathematics by engaging students with mathematics in creative and diverse ways (Nemirovsky et al., 2017).

Regarding gender difference, the negative impression of mathematics have affected girls more than boys (Goldman & Booker, 2009). The purpose of this study is to investigate the images of mathematics from a group of women who had participated Girls Excelling in Math and Science (GEMS)–a girls only STEM afterschool program 26 years before. The two research questions below guide this study:

1. What similarities or differences are in the participants' experiences with mathematics in the classroom and GEMS?
2. How do participants' experiences in both school and informal learning environments impact their current image of mathematics?

In this case study, we investigated nine participants' described experiences in school and GEMS. The data source includes survey, focus group interviews, and individual in-depth interviews. In addition, observation, research memo and artifacts are used as secondary data source. We utilized thematic analysis (Braun & Clarke, 2012) to identify common themes across participants' descriptions. The preliminary findings show that OGGs image of mathematics in the informal environment was positive compared to their image of mathematics in the classrooms. Although their images of mathematics were mediated by informal learning experiences in some extent, their general impressions of mathematics are greatly influenced by their school mathematical experiences

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# Preservice Teachers' Use of Mathematical Knowledge for Teaching During Lesson Planning

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Planning a lesson, including formulating mathematical goals and constructing purposeful questions is an important mathematics teaching practice (National Council of Teacher of Mathematics [NCTM], 2014). Lesson planning includes teachers' decisions related to structuring content that is relevant for a particular group of students, suggesting that teachers need to address the non-linear nature of student reasoning during lesson planning (Davidson, 2019; Lai & Lam, 2011). To address students' reasonings, mathematics teachers need to have discipline-specific knowledge, which assists them in formulating appropriate mathematical goals and selecting purposeful questions to address students' mathematical thinking (Ball et al., 2008; Ribeiro et al., 2013). In mathematics education, the discipline-specific knowledge required for teaching is commonly known as Mathematical Knowledge for Teaching (MKT; Ball et al., 2008). MKT has two domains (Subject Matter Knowledge & Pedagogical Content Knowledge) and has the following six subdomains (referred to as "MKT domains"): Common Content Knowledge, Specialized Content Knowledge, Horizon Content Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum. Mathematics teacher educators have suggested the influences of MKT domains to foster both in-service and preservice teachers' (PSTs) lesson planning skills (e.g., González et al., 2020). Studies have also indicated that both in-service teachers and PSTs demonstrated the growth in their MKT after constructing, discussing, and reflecting on lesson planning (e.g., Shuilleabhain, 2016).

Despite these connections between teachers' lesson planning skills and MKT domains, which MKT domains secondary mathematics PSTs (M-PSTs) demonstrate during lesson planning is still underexplored. Indeed, the studies have suggested M-PSTs struggle to utilize the advanced mathematical knowledge learned from content courses in their teaching (e.g., selecting instructional strategies; Ball & Bass, 2003; Wasserman et al., 2019). In this study, I investigate how secondary M-PSTs utilize MKT during lesson planning. The following research question guides this study: How do secondary mathematics preservice teachers (M-PSTs) demonstrate their use of MKT domains during lesson planning.

## Methods: Participants, Data Sources, Methods of Analysis

All the M-PSTs enrolled in a secondary mathematics methods course at a large midwestern university participate in the study. I facilitate a series of instructional activities involving lesson planning, reflecting on planning, implementing lessons, and reflection on lesson implementation in the methods course. Documents collected from M-PSTs' lesson plans will be the data for the study. Utilizing a multiple case-study design, I compare how M-PSTs utilize six MKT domains during lesson planning (Yin, 2017). I use the Verbal Analysis Method to analyze my data, which allows researchers to use a set of codes driven by theory (top-down approach) and iteratively refine codes using the data (bottom-up approach) (Chi, 1997). I begin my coding process using the codes that I developed from the MKT framework (Ball et al., 2008). I iteratively revise this coding scheme from multiple rounds of coding (Schreier, 2012). Finally, I report the findings by identifying the themes from the coding process.

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# Examining students' understanding of numbers and operations using an online number sense three-tier test


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**Abstract**

*Numbers and operations typically occupy a large time of instructions and learning contents in elementary schools, given its importance in providing the fundamental mathematics concepts and skills. This study documents 372 fifth-grade students' understanding of numbers and operations when responding to an online three-tier number sense test, which examined content knowledge (i.e., questions and answers), explanatory knowledge (reasoning/reasons), and confidence level regarding the answers and reasons.*


One of the core components of mathematics teaching and learning in elementary schools is developing students' number sense (National Council of Teacher Mathematics, 2000; Şengül & Gülbağcı, 2014). An individual with sufficient understanding of numbers and operations typically exhibits good number sense and vice versa (Şengül & Gülbağcı, 2012; Yang & Sianturi, 2021). However, many students remain exhibiting related misconceptions even at the higher education levels (Merenluoto & Lehtinen, 2002; Sengül & Gülbağcı, 2014). Notably, originated in psychology, confidence ratings have been used as a promising predictor of students' conceptual understanding and related misconceptions (Caleon & Subramaniam, 2010; Pesman & Eryilmaz, 2010; Yang & Sianturi, 2019), which could be identified by measuring the certainty of responses. This study is aimed to reveal the current performance, misconception, and confidence level of students. The research questions were as follows:

(1) How is fifth-grade students' understanding of numbers and operations?

(2) What are some strong misconceptions held by the fifth-grade students?

## Theoretical Framework

### Number Sense and Components of number

Number sense can be defined as the ability to apply knowledge and understanding of related concepts in numbers and operations domain using flexible and efficient strategies to solve numerical problems (Berch, 2005; McIntosh, Reys, & Reys, 1992; Şengül & Gülbağcı, 2014). Five crucial components of number sense (Table 1) were established through the review of experts in the field and its alignment with the mathematics curriculum and learning trajectories in elementary schools.

## Methods

**Study participants:** 372 fifth graders with varied socioeconomic backgrounds from Malaysia participated in accord with the guidelines outlined by an institutional review board.

**Study instrument:** An online Three-Tier Number Sense Test (TTNST) comprising 40 questions was established to examine students' understanding of numbers and operations and their confidence levels when responding to the test items.

**Procedure, data collection, and analysis:** This study focuses on an application of the online three-tier test previously developed by the author (Yang, 2019). To analyze students' responses, a scoring rule (Table 2) was developed by modifying the scoring rules used in earlier studies on a three-tier test to fit the context of this study. To reveal a significant misconception, a criterion on the basis of incorrect responses and confidence levels was established: 16.25% = 1/16*100% (given 16 answers for a question) + 10%; 10% is a threshold suggested in earlier studies (Caleon & Subramaniam, 2010; Yang & Sianturi, 2019). A strong misconception in this study is a significant misconception with a mean confidence level higher than 3.5 (high confidence).

**Reliability and validity:** This study is an application of an assessment instrument developed for assessing students' understanding of numbers and operations and their number sense (Yang, 2019)—the instrument has been proven to have satisfactory reliability and validity.

**Preliminary Findings:** Most students exhibited unsatisfactory performance on the test (low number sense). Only 18.28% of them had a profound understanding of related concepts in numbers and operations, while 77.96% had strong misconceptions. Most students had difficulties in judging the reasonableness of computational results and in solving context-based problems.

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## Appendix

Table 1. Five most pivotal components of number sense used in the study

| Component | Explanation |
|---|---|
| Understanding the basic meaning of numbers and operations (C1) | This component involves an understanding of the number system (e.g., whole numbers, fractions, decimals, place value, number patterns, and the use of multiple ways to represent numbers) (Berch 2005: McIntosh et al., 1992; Purnomo et al., 2014). |
| Recognizing the number size (C2) | This component involves recognition of the relative size of numbers. For instance, students should be able to use meaningful ways to solve the problem when comparing fractions with the same numerator, same denominator, transitive, and residual without relying on standard written methods (e.g., finding the least common denominator) (McIntosh et al. 1992, 1997; Purnomo et al., 2014; Yang, 2019). |
| Using multiple representations of numbers and operations (C3) | This component involves the ability to present, use, and switch among different representations and the most appropriate representation (Alajmi & Reys, 2010; McIntosh et al., 1992; Yang, 2019). For example, students should know that $\frac{1}{2}$ could be represented in different representations or forms (e.g.,$\frac{4}{8}$, 50%, 0.5, or using visual and symbolic representation). |
| Recognizing the relative effect of operations on numbers (C4) | This component involves the ability to make sense of the operations (McIntosh et al., 1992, 1997; Purnomo et al., 2014). For example, multiplication can increase or decrease a value of fractions. |
| Judging the reasonableness of a computational result via different strategies (C5) | Students can mentally apply different strategies to solve problems without using written computation (Alajmi & Reys, 2010; McIntosh et al. 1992; Yang, 2019; Yang & Sianturi, 2019). For instance, when students were asked to answer a question, *"Please place the decimal point correctly using estimation(s)": 938.5 × 0.496 = 465.496; (a) 46.5496, (b) 465.496, (c) 4654.96, (d) Answer could not be found,* they do not need to rely on using paper and pencil or recalling mathematical rules; instead, they should be able to think that the product of 900 × 0.496 (about 1/2) is about 450; therefore, (b) 465.496 is a reasonable answer. |

Table 2. Scoring rules applied to analyze students' responses to the TTNST

| **Number sense three-tier test** | | | | | | |
|---|---|---|---|---|---|---|
| 1st stage | *Answer* | Correct | | | | Incorrect |
| | Score | 4 | | | | 0 |
| 2nd stage | *Reason* | NS-based | Rule-based | Misconception | Guessing | 0 |
| | Score | 4 | 2 | 1 | 0 | |
| **Total score** | | **8** | **6** | **5** | **4** | **0** |
| | *Confidence Rate Index (CRI)* | | | | | |
| 3rd stage | CRI | Very confident | Confident | Neutral | Unconfident | Very Unconfident |
| | Score | 5 | 4 | 3 | 2 | 1 |

*Note.* NS-based: Number-Sense based method; Rule-based: Rule-based method; If a student answered correctly with a NS-based method, the student would be given 8 points, and so on; conversely, an incorrect answer was given 0 point, regardless of the reason selected for the answer.

**Core Practices in the Context of an Elementary Mathematics Methods Course**

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**Abstract**

*Paired with learning theory, core practices can help pre-service teachers gain practical knowledge that they can use to effectively implement instruction. This self-study looks at how the core practices of organizing class discussion and eliciting student thinking are represented, decomposed, and approximated in the context of an elementary mathematics methods course.*

**Purpose**

The purpose of this self-study is to investigate the researcher's process in learning to teach two core practices to undergraduate elementary education students in a methods course.

**Literature Review**

There are several synonyms for the term, “core practices”. Grossman and her colleagues (2018) as well as others (Thames & Van Zoest, 2013) have attempted to provide a standard set of vocabulary to talk about core practices, the language is not standardized across the field. As defined by Grossman et al. and the Core Practices Consortium (CPC), core practices are “identifiable components fundamental to teaching that teachers enact to support learning. Core practices include both general and content-specific practices and consist of strategies, routines, and moves that can be unpacked and learned by teachers” (Core Practices Consortium, 2020; Grossman et al., 2018, p. 4). Some research uses “core tasks” interchangeably with “core practices” (Ball & Forzani, 2009). While CPC has chosen to adopt the “core practices” language, other organizations and researchers have chosen to use “high-leverage practices,” which Grossman positions as another name for the same concept. For example, TeachingWorks calls high-leverage practices “a core set of fundamental capabilities” and lists 19 specific practices on their website (TeachingWorks, 2020). Another name for core practices is “ambitious teaching,” which is common among mathematics core practices research publications and has been defined as a “group of related concepts and models of instruction” (Lampert & Ghousseini, 2012, p. 7).

Among these various lists of core practices, two are common: leading discussion and eliciting student thinking. Organizing productive discussion appears to be a challenge for many teachers, partly because it is difficult to know how students are thinking about a mathematical task and then use that information to structure a student discussion (Ghousseini, 2015; Smith & Stein, 2011; Tyminski et al., 2014).

**Research Questions**

The questions this research will investigate are:

1. What are we (the course instructors) doing that appears to support pre-service teachers' skills in eliciting student thinking and orchestrating productive mathematical discussions, despite the constraint of having no field experience available due to COVID, which limits the ability to practice?

2. Where do I see myself in the practice of teaching pre-services teachers these core practices?
3. How are the core practices of organizing discussion and eliciting student thinking represented, decomposed, and approximated over the course of the semester?

**Methodology**

As I teach a section of an elementary mathematics methods course, all curricular materials will be collected and analyzed for their relevance to organizing class discussion and eliciting student thinking. I am keeping a journal about course meetings, student work, and my reflections on them to review and discuss every other week with my critical friend, a professor in the Department of Curriculum and Instruction at Indiana University. I am also keeping track of weekly comments by students that have been elicited through a form specifically for gathering their thoughts on the course as we move through the semester, and these will be analyzed to look for ties to the core practices being studied.

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# Understanding an Undergraduate's Units Coordination

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## Abstract

*Units coordination and spatial reasoning have been investigated with elementary and middle school students to support better instruction in those areas, as they can impact future mathematical learning in higher grades. But what happens when a student reaches college without having being able to coordinate three levels of units and without a robust conceptual understanding of fractions? This teaching experiment explores such an undergraduate's thinking and looks for mathematical tasks that might support the development of more sophisticated understanding of units coordination and fractions for that student.*

## Purpose

The purpose of the study is to investigate the units coordination stage and fractional understanding of an undergraduate who struggles in mathematics as well as to design and test mathematical tasks that would support progression and deeper understanding of units coordination and fractions.

## Literature Review

The coordination of units across multiple levels is needed for students to be able to work across multiple mathematical domains, including multiplication and fractions (Hackenberg, 2013; Hackenberg & Tillema, 2009) As students move through upper elementary and into middle school, they are expected to understand how to work with fractions, negative integers, and algebraic notation. It is well-documented that students struggle with these topics and that these areas of learning are impacted by students' understanding of number operations (Steffe & Olive, 2009; Ulrich, 2016). There has been some research that suggests that paying attention to students' spatial structuring is related to how they coordinate multiple levels of units (Lowrie et al., 2016, 2017; Mix et al., 2020; Ulrich & Wilkins, 2017).

While most research has focused on elementary and middle school students, little research has been completed with undergraduate students who still show lower levels of units coordination, which may impact their ability to complete college-level mathematics courses.

## Research Questions

The three questions this research will investigate are:

1. What is the student's stage of units coordination and fractions knowledge?
2. What spatial structuring do we see as we uncover the student's units coordination and fractions understanding?
3. What mathematical tasks and interactions appear helpful in moving the student towards more sophisticated units coordination and fractions knowledge?

## Methodology

The study is structured as a teaching experiment with an undergraduate elementary education major who has demonstrated difficulty in working with fractions at a conceptual level. The student will be given a previously-validated assessment to determine their units coordination at the beginning and end of the study (Norton et al., 2015). In a series of meetings, the student and researcher will work on various mathematical tasks to probe the students' thinking and to push them toward a deeper understanding of units coordination and fractions, while paying attention to how the student visually organizes their thinking. Some of the tasks come from prior work done with middle schoolers (Hackenberg, 2013) and others will be designed based on interactions with the student.

Meetings with the student will be conducted and recorded over Zoom. Recordings of student work as it is produced will be made through a variety of technologies, such as Google Jamboard, video recording of paper-based work, and Fraction Bars software. Conversations with the student will be analyzed to look for effective ways of assessing and advancing their thinking.

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## Examining Tall's Worlds in Curriculum

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**Abstract**

*Multiple representations of mathematical objects play a key role in undergraduate linear algebra courses. These representations may be classified in terms of Tall's worlds of mathematics: the embodied world, the symbolic world, and the formal world. Harel has examined how the embodied world appears in textbooks and the purposes for which it is used. In this study, I will extend this work by qualitatively analyzing how each of the three worlds appears in the intended, enacted, and attained curricula by examining selected portions of the textbook, lectures, and student work from a linear algebra course. Preliminary results from the textbook analysis indicate that the embodied world is used primarily for generalizations and literal representations. Beyond definitions, theorems, and proofs, the formal world is used for non-proof explanations, preventing misconceptions, and introductions. The symbolic world has the widest variety of uses, including all the ways the formal world is used as well as providing computational procedures and connecting the material to computational issues with technology. The ways worlds are used to introduce concepts is not constant. When introducing linear independence, the textbook moved from the symbolic world to the embodied world. The introduction of span started and ended in the embodied world, with the symbolic world in between. Further analysis will refine these results and determine whether the worlds appear in similar ways and are used for similar purposes in the intended, enacted, and attained curricula.*

Multiple representations of mathematical objects are crucial in linear algebra. Representations may be classified in terms of Tall's (2013) worlds of mathematics: the embodied world, the symbolic world, and the formal world. The embodied world contains graphs and geometry; the symbolic world, operations and algebra; and the formal world, set-theoretic definitions and proofs. Researchers have examined difficulties that varying worlds pose for students and ways that students think within different worlds (e.g., Hillel, 2000; Dogan-Dunlap, 2010; Sandoval & Possani, 2016; Sierpinska, 2000; Tall, 2013). One potential source of difficulty is the curriculum. Researchers have called for studies examining the relationship between curricula and student understanding (Fan et al., 2013).

In this study, I analyze how Tall's worlds appear and are utilized in the intended, enacted, and attained curricula (see Lloyd et al., 2016) of a linear algebra classroom. Researchers have not examined how the symbolic and formal worlds appear within the curriculum. Harel (2019) has analyzed the purposes for which the embodied world is used in linear algebra textbooks. After looking broadly at the purposes for which each world is used, I focus on the worlds presented and their purposes surrounding the topics of linear independence and span. This study could reveal ways curriculum influences students' usages of mathematical representations and ways to modify curriculum to aid students with effectively using different representations.

My data consists of the textbook, lectures, and four students' work on homework assignments within one linear algebra course. My research questions are:

1. How are Tall's three worlds of mathematics presented and utilized in the intended, enacted, and attained curricula?
2. How are Tall's three worlds of mathematics presented and utilized in the intended, enacted, and attained curricula on the topics of linear independence and span?

Data were assigned two codes: the first, one of Tall's worlds; the second, the way the world is used. Coding initially took the form of a line-by-line analysis. After coding, I will compare the purposes of the data across the worlds of mathematics and the types of curricula. Then I will focus my analysis on the topics of linear independence and span to determine the usage and purposes of each world on the scale of a single concept.

Preliminary results from the textbook analysis indicate the embodied world is used primarily for generalizations and literal representations. Beyond definitions, theorems, and proofs, the formal world is used for non-proof explanations, preventing misconceptions, and introductions. The symbolic world has the widest variety of uses, including all the uses of the formal world as well as providing computational procedures and connecting the material to computations using technology. When introducing linear independence, the textbook moved from the symbolic world to the embodied world. In contrast, the introduction of span started and ended in the embodied world, with the symbolic world in between. In further analysis, I will refine the results presented here and determine whether the worlds appear in similar ways and are used for similar purposes across the intended, enacted, and attained curricula.

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**A Comparative analysis of Congruence in U.S and Chinese Standards and Textbooks**

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Congruence is an essential concept in geometry. “Students should learn to use deductive reasoning and more-formal proof techniques to solve problems and to prove conjectures [through learning congruence]” (NCTM, 2000, p. 42). Study reveals that U.S. and Chinese curriculum use different approaches to define congruence and address the topic differently (Zhou et al., under review). This study aims to examine the topic of congruence in the U.S standards and Chinese mathematics standards and textbooks. We investigate Common Core Mathematics State Standards (CCSSM) which is widely adopted in the U.S and Chinese Compulsory Education Mathematics Curriculum Standards (CMCS) which is the only national standards for elementary and middle school in China. For comparison, we selected the widely used Chinese textbook for Grade 8, published by People Education Press, which is aligned with CMCS, and the U.S. textbook for Grade 8 – Eureka Math, published by Great Minds, which is aligned with CCSSM. The previous study reveals that CCSSM uses the transformation approach to develop dynamic definitions of congruence while CMCS uses a traditional approach to develop static definitions of congruence (Zhou et al., under review). Historically, there are many debates around the strength and weakness of these two approaches of definitions of congruence (Sinclair, 2008). The debates motivate us to investigate how the two textbooks proceed these two approaches.

The present study is guided by two overarching research questions. First, how CCSSM and CMSC present the topic of congruence and what learning expectations are proposed in these two sets of standards. Second, how the two textbooks proceed the concept of congruence and congruent triangles. In particular, the standards comparison aims to investigate the philosophy of geometry behind the topic of congruence by inquiring how congruence connects with other geometrical topics crossing grade levels or grade bands. The textbooks comparison focuses on the following three dimensions of analysis: how the topic is introduced, what examples/problems are provided, and how the contents progress students’ geometric proof.

The ongoing analysis indicates that both textbooks refer to congruence as same size and same shape. However, in line with the standards in both countries, the two textbooks present congruence in different ways. In the U.S textbook, the concept of congruence is defined by rigid motions (i.e., translations, reflections, and rotations). In the Chinese textbook, the concept of congruence refers to all corresponding pairs of sides, and all corresponding pairs of angles are equal. Moreover, regarding the topic of congruence, the structures and contents of the two textbooks are distinctive. The U.S. textbook uses module structure and includes four sub-topics: definitions of the basic rigid motions, sequencing the basic rigid motions, congruence and angle relationships, and the Pythagorean theorem which is optional. The Chinese textbook uses chapter structure and consists of three sections: definition of congruence, the criteria of congruent triangles, and bisect of an angle. The findings will reveal more details about the topic of congruence in two standards and two textbooks. We will further our discussion on geometry curriculum including standards, textbooks, and instructions.

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# Chapter 4: Work under-Design

# Teachers' care behind the stage: Teachers’ curricular decisions

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## Abstract

*Mathematics builds on itself, but this cumulative creation in students' minds requires careful selection of problems. Multiple studies linked teachers' conceptions, skills, and goals to curriculum decisions, less research explores teachers' care concerning curriculum decision making. This case study investigates how teacher's care influences curricular decisions. Data from 10 teachers consisting of a questionnaire, interview, curricular resources considered and selected will be analyzed using the caring mathematically framework. Findings in the form of correspondence between traditional factors that influence teachers' curricular decisions and ways teachers harmonize with the students' thinking to analyze the curricular used will be constructed. These findings support the argument that teachers’ care is a factor in their curricular decisions.*

## Introduction

We can think about a classroom environment as a stage because teachers' and students' have roles that intertwine acting in the learning process. Many stories take place on the classroom stage. Of all the stories, I picked the story of caring. Teachers' care for the students extends beyond the classroom. I decided to look at teachers' work and care behind the stage because it is equally valuable as their work and care in the classroom. This paper focuses on teachers' care outside the classroom stage, during the prep time, or any other moment when they reflect and decide about curricular resources considered and selected for the students. I want to determine how teachers coordinate the curricular decisions towards harmonizing the students' work and interests.

The National Council of Teachers of Mathematics stated that students gain fluency in mathematics if teachers carefully select the problems ([NCTM], 2014). Teachers' interaction with curricular resources directly influences students' learning mathematics. When teachers attend and analyze the students' cognition, they find ways to respond to the students' thinking, and their responses is materialized in curricular selection. Like prescriptions, these selections put students in mathematics problem-solving situations. Through problem-solving selection, they try to engage the students. Student thinking shared during problem-solving allows teachers to notice the students' thinking. Attending and analyzing students' cognition is essential for creating mathematical caring relations (Hackerberg, 2010).

## Research Question

The research question for this study explores how teachers’ care influence curricular decisions?

## Methods and Participants

A questionnaire, followed-up by an interview, and curricular resources selected will be analyzed. Findings that link traditional factors that influence teachers' curricular decisions with caring factors that harmonize with the students' thinking and interest will be considered for the analyzes. The participants for this study in number of 10 will be selected based on convenience.

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# Complexity Theory and Virtual Education. What is it changing?

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## Abstract

*This proposal corresponds to a research process in its initial state, i.e. work under design. Firstly, a rational to investigate virtual mathematics classroom is defined. Secondly, the study considers a literature review in relation to the Complexity Theory in education, particularly, considering the five conditions to understand the classroom as a complex system (Davis & Simmt (2003): internal diversity, redundancy, decentralized control, organized randomness, and neighbor interactions. Lastly, research questions and possible methods are proposed.*

## Rationale

With COVID-19 challenging educative environments, the *Mathematics classroom* is evolving toward innovative teaching methods and new ways of learning. Therefore, *Mathematics teacher educators* are understanding new educative paradigms as well as redefining their own practices in the virtual classrooms. The complexities around how the process of teaching is developed, and how people interact among themselves seem to be critical features to redefine new educational practices. Thus, understanding the complex aspects of virtual environments will allow well-situated teaching and learning situations.

## Literature Review

The notion of complexity has emerged from a transdisciplinary work within the application of complex systems to different fields (Cilliers, 1998). In the Education field, Davis & Sumara (2006) proposed a theoretical framework to understand the complexities around the Mathematics Classroom. Particularly, Davis & Simmt (2003) illustrate the complexity of a Mathematics community throughout five principal conditions of a complex system: *internal diversity, redundancy, decentralized control, organized randomness,* and *neighbor interactions.* While these five conditions were described within in-person education, the new virtual educative environments open new windows to observe how the aspects of complexity are changing, and thus, aiming for new explanations and resources to redefine the virtual Mathematics classroom.

## Research Questions

The study will partially draw answers toward the following questions:

- How can the five conditions be characterized in virtual environments?
- How do the five conditions differ between in-person and virtual activities?

An ultimate question can also guide this proposal from a general perspective:

- Is a virtual educative environment a Complex System?

## Proposed Methods

The context should be a virtual mathematics classroom, although the level is not confirmed at this point. At least, the study should consider a description of interactions within the class. Interviewing participants and/or describing a class may be options to conduct the study.

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**Angle your laser: Technological exploration of using radians in a STEM context**

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Despite the increased demand for integrated science, technology, engineering, and mathematics (iSTEM) education since project 2061 (e.g., American Association for the Advancement of Science, 1989), the issue of unequitable representation of STEM disciplines in iSTEM education remains. Mathematics education specifically is underrepresented in iSTEM education, and there is an increased need for research and curricular approaches that could give rise to mathematics in an iSTEM context (English, 2016). In this study, I intend to address this issue, by exploring secondary preservice teachers' (PSTs') ways of thinking of a mathematical concept in a science context. Specifically, the concept of radian angle measure embedded with the science of light reflection. This study is guided by the research question "What ways of thinking about angle measure are expressed by PSTs upon interacting with a digital curricular situation that involves radian angle measure with light reflection?"

## Methods

To explore PSTs' ways of thinking about radian angle measure, I utilize Desmos, a website-based technological tool that is well known for its free online graphing calculator website. However, Desmos also provides a variety of free digital classroom activities where learners can explore mathematical concepts in a deeper way (Desmos, 2020). Utilizing a digital activity through Desmos aligns with NCTM's (2000) recommendation for learners to "develop a deeper understanding of mathematics with the appropriate use of technology" (p. 24). I build on an original Desmos activity (i.e., designed by Desmos) titled *Laser Challenge* (Figure 1). I will use a shorter version that I have adapted, *Laser Challenge (Radian)*, which I developed so that the input angle measure must be in radian. The task works as a game with immediate feedback where the user can identify if their answer is correct once the laser passes simultaneously through the three targets.

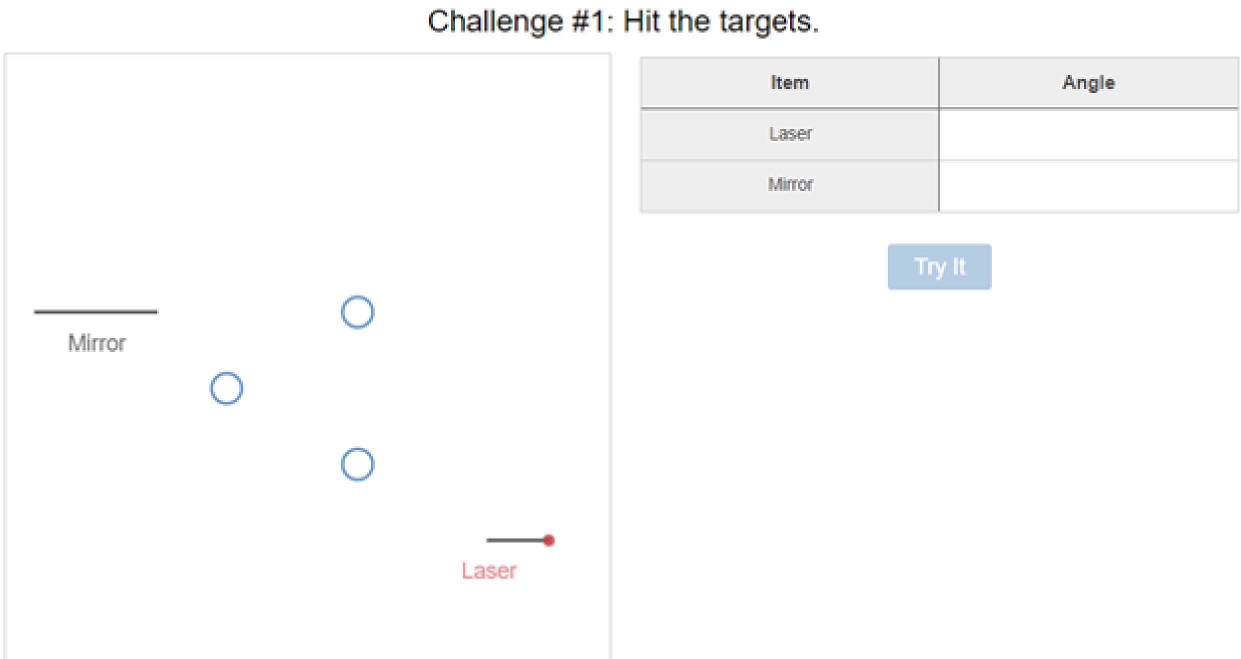


*Figure 1.* Example of the laser challenge activity.

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**Exploring the combinatorial reasoning in undergraduate students' classrooms**

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**Abstract**

*The study proposed in this document addresses the teaching and learning of combinatorics; particularly, its basic counting strategies and formulas. As different researchers have studied individual student's reasoning and learning of combinatorics, this proposal aims to analyze the development of combinatorial thinking when is being studied in the context of a regular undergraduate course.*

## Rationale of the study

Combinatorics is a powerful area of mathematics that offers not only interesting and challenging problems; but also, as Lockwood (2014) describes, offers a rich opportunity for students to develop their problem–solving skills, critical mathematical reasoning, justification and modeling abilities, among others. The aforementioned quality and its existing connection with other areas such as computer science, algebra, and probability, provides sufficient reasons for studying this fascinating area of mathematics.

## Literature review

The learning of combinatorics has been a topic of interest in different countries. In Spain, studies such as Batanero, Godino, Roa (2003) and Roa, Batanero, Godino, (2001) present strategies and difficulties evidenced in students when solving combinatorial problems. Similarly, in the US, studies such as Lockwood (2012) and **Lockwood,** Swinyard, Caughman (2014), respectively present a model of the students combinatorial thinking and an analysis of the combinatorial thinking of two undergraduate students as they work with basic counting problems and reinvent basic combinatorial formulas. The interest presented in this proposal is to study the combinatorial thinking of undergraduate students in the context of an average classroom.

## Research Questions

1. How is the development of the combinatorial thinking of a group of undergraduate students as they study combinatoric through solving basic counting problems and reinventing combinatorial formulas?
2. How is the development of the student's individual combinatorial thinking as they participate in classroom study of combinatorics?

## Proposed Methods

The study could be developed under Cobb's (1996) Emergent Perspective research design. Involving: (a) The execution of class sessions with the whole group of participants, which will provide an opportunity to witness the emergence and evolution of socio-mathematical norms and classroom mathematical practices; (b) The conduction of individual interviews that allow to describe the mathematical conceptions of each participant involved in the study.

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**Impact of units coordination in reminding fractional knowledge for high school students**

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## Abstract

*In my experience teaching math for high schoolers, I had many students having difficulty in calculating fractional numbers, even though they learned fractions since they were in elementary school. In order to help those students, teachers need to lead their students to consider fractions as quantity. In this regard, units coordination can be a solution to do that with partitioning, disembedding, and iterating. In this proposal, I would like to show that students with a lack of fractional knowledge can improve and robust their fractional knowledge by learning partitioning, disembedding, and iterating as units coordination.*

## Rationale

As teachers face students who have difficulty in the operation of fractions knowledge in high school, there are needs for a useful tool of reminding fractional knowledge. From my literature review, I attributed learning units coordination can help students understand fractions knowledge deepens. Thus, the purpose of this study is to investigate how high school students can develop their fractions knowledge by learning units coordination.

## Literature Review

*Fractions as quantity* can be a start for students to regard fractions as visualization so that they can figure out mathematical reasoning in fractions knowledge. When students are asked to draw fractional amounts of units of measure that are represented by segments or rectangles, fractions may be considered as measurable extensive quantities (Hackenberg, 2010).

*Units coordination* can be helpful for students to interiorize their fractions knowledge by visualizing fractions as well as imagining fractions with mental actions. The construction of improper fractions requires students to have the *interiorization* of three multiplicative concepts (Hackenberg, 2007).

*Operations* are considered as mental actions in conceiving subdividing a unit into equal parts (Piaget, 1970; von Glasersfeld, 1995). For the fractional knowledge, operations are *partitioning* of a unit, *disembedding* one of the partitioning parts, and *iterating* as many as we need.

## Research Questions

1. What are students' levels of units coordination?
2. How units coordination can contribute for students to learn fractional knowledge?

## Methodology

In order to conduct this research, I will conduct qualitative research. At first, a written fractions assessment will be given so that researcher is able to estimate students' level of units coordination as well as select interview participants in order to observe students with difficulty in fractional numbers. After then, a couple of interviews will be conducted with the concept of units coordination.

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# Exploring Pre-Service Teachers' Lesson Plans to Promote Improper Fractions

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## Abstract

*Splitting operation is an essential skill to produce improper fractions. Students can have splitting operations with/without interiorizing three levels of units. Therefore, pre-service teachers can be able to design lesson plans involving different tasks for all students. In this study, we will explore pre-service teachers' pedagogical content knowledge by analyzing their lesson plans on improper fractions.*

## Introduction

Pedagogical Content Knowledge(PCK), which refers to interpretation and transformation of subject matter knowledge, includes understanding students' conceptions (Schulman 1987). Fraction knowledge such as improper fractions is difficult to learn and teach (Hackenberg, 2007; Kerslake, 1986; Newton, 2008) because teachers have limited content knowledge and pedagogical content knowledge (e.g., understanding students' conception about fractions). Therefore, most researchers recognized pre-service and in-service teachers' lack of content knowledge (e.g., Tirosh, 2000;Newton, 2008; Putra,2019) and pedagogical content knowledge (e.g., Naiser et al. 2004; Isiksal & Cakiroglu, 2011) about fractions. For example, Naiser et al. (2004) observed teachers' pedagogical content knowledge over 4 months and described ways to improve teaching fractions, i.e., understand students' initial concepts, using manipulatives. However, little research has been done on pre-service teachers' task design skills on complex fractions concepts such as improper fractions. In this study, we will investigate pre-service teachers' tasks to promote students' conception of improper fractions.

Some researchers explored students' construction of improper fractions (e.g., Hackenberg, 2007; Steffe, 2002). Steffee (2002) argued that an iterative fraction schema is not sufficient to independently produce improper fractions; therefore, splitting is an essential multiplicative operation to construct improper fractions. Splitting operation is accompanied by interiorizing three levels of units (Steffe, 2002). However, Hackenberg (2007) showed that students can have splitting operations without interiorizing three levels of units. Therefore, teachers can be able to design different mathematical tasks for students who interiorized three levels of units than for students who did not interiorize three levels of units (Hackenberg, 2007). In this study, we prompt PSTs to design a lesson plan that includes two different tasks for students who did/did not interiorize three levels of units to promote. Therefore, we will explore the following research question:

1. How do PSTs design a lesson plan to foster students to produce improper fractions? (a) for students who interiorized three levels of thinking, (b) for students who did not interiorize three levels of thinking.

We will ask PSTs to design a lesson plan to help students to produce improper fractions. Then, we will analyze their lesson plan to explore how they facilitate producing improper fractions.

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# Going Beyond Gender-Complex Education: A Study of Sexual Orientation in Mathematics Education

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**Abstract**

*This study pushes back on the teaching of mathematics for social justice and studies that ascertain issues of gender overlooking issues of sexual orientation. Although the author sees an extreme importance of the teaching of mathematics for social justice, and this is the author's ultimately desires (that the teaching and learning of mathematics always help students to challenge social issues), it's hard to know and write about what is effective and what will produce positive engagement towards learning, if students from these different genders and sexual orientations aren't consulted at first. I am not arguing that no research has been done by mathematics educators in trying to understand transgender and gender-nonconforming students' experiences, but there is a trend after Rands (2013) publication on Add-Queer-And-Stir and Mathematics Inque(er)y that have resulted in multiple investigations of how to queerize the curriculum. Interestingly, little in this new realm of research has considered the students' experiences. Rather, they suggest what can be done to make the classroom more inclusive to LGBTQ+ students or dismantle a variety of normativities that come in mathematical problems. Because of the complexity of the LGBTQ+ groups, and having in mind that such an acronym refers to gender identities, sexual orientation, and sexual identities, research has also overlooked the sexual orientation part of these groups. Leaving gays, lesbians, bisexuals, asexuals, demisexuals, and graysexuals excluded from important conversations that must be challenged as mathematics continues to propagate heteronormativity. In addition, not respecting and understanding how we can in fact impact learning for LGBTQ+ students in ways that we are going forward creating safe learning spaces, in fact, for these students. Thus, this proposed study will investigate how LGBTQ+ students respond to issues of sexual orientation being discussed in the mathematics classroom.*

Studies in mathematics education have attested that the field of mathematics is white and masculine (Leyva, 2017). More recent research has shown that mathematics goes beyond just being white and masculine, but the field is also heteronormative and cisnormative (Waid, 2020). As much as people of color and women are not part of the field, historically marginalized students that belong to LGBTQ+ groups are also not part of the field (Greathouse et al., 2018). Since research has shown that valuing students' identities through mathematics supports students' identity development as mathematics doers (advancement in learning) and consequently mathematics choosers (Aguirre, Mayfield-Ingram & Martin, 2013), the need to bring more diversity to mathematics, more studies on identity support and development is becoming more imperative. To bring change for mathematics and help mathematics classrooms function as safe spaces for LGBTQ+ students and spaces that educate about LGBTQ+ lives (advancement in instruction), this study proposes an intervention through mathematics in high school classrooms by going beyong a *gender-complex education*. Rands (2013) defines *gender-complex education* as implementing mathematical curricula and pedagogy that acknowledges gender diversity. Rubel (2016) extends *gender-complex education* theoretical underpinnings to discuss the need to examine ways mathematics tasks (re)presents gender and sexual orientation norms. This study will use mathematical problems that elicit and create discussions in mathematics that directly address issues of heteronormative. Design-based research methodology (Cobb, 2001) will be used and through teaching experiment (Steffe & Thompson, 2000) data will be collected. I will investigate how LGBTQ+ high school students respond to certain mathematical problems and, at the same time, have the chance to evolve/explore their understandings of their sexual orientation identities. Finally, follow-up interviews will be used to understand how the teaching episodes impacted ten high school students. This research intends to respond to the following research questions: 1) How does a teaching experiment using a *gender-complex education* framework impact LGBTQ+ students sexual orientation identities? 2) How do LGBTQ+ students responds to *gender-complex education* usage in the mathematics classrooms?

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# Exploring the Impact of Mathematics Problem Solving as Informal Play in the Time of COVID -19

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**Abstract**

*The curiosity for mathematics is delightful in informal settings. This research explores people's involvement in mathematical play activity during COVID-19 quarantine. An easel placed near the sidewalk in an urban subdivision showed mathematics problems during the COVID-19 pandemic. People embraced with joy this social activity in the form of mathematics play during the quarantine. Interviews with eight participants in the mathematical activity will reveal what they have experienced and how they experienced the mathematical activity as adults after they have finished school.*

## A feel of connectedness through mathematical play

With the school closure for the pandemic on March 13, 2020, many responsibilities that teachers had, fell on parents' shoulders. Parents looked for various ways to entertain their children trying to make pandemic time as normal as possible. We thought that posing mathematics problems in the form of play for now *stay-at-home students* would add entertaining and connectedness, transmitting our care for their well-being (Tull et al., 2020). To our surprise, many adults enthusiastically embraced the mathematics activity, engaging in solving problems with their family members. Through collaboration (Martin et al., 2006), they come up with solutions.

An easel placed in the proximity of the sidewalk showed mathematics problems in the time of Covid-19 between April and October 2020. Around 20 problems were written with chalk on an easel. The participants are inhabitants of an urban subdivision in North of Indiana in the United States. In this study, we want to find out what people experienced and how it was experienced. The study will be a phenomenological exploration that focuses on individuals' lived experiences during the Covid-19 pandemic when involved in informal mathematics activities. Eight adults, excluding mathematicians, will be interviewed. The purpose of the study is to respond to the following research questions:

1. What benefits do adults gain by doing mathematics when they are no longer students?
2. How did this activity help their families somehow during the Covid12 pandemic?

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